

# Performance Evaluation of LDPC Codes With Mackay Matrix

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**Abstract:** The main goal in designing a communication system is to achieve reliable data transmission with as small a transmission power as possible, in other words, a power efficient system with the lowest error probability (bit error rate or frame error rate). Moreover, a higher data rate with a constraint on available bandwidth is another target. LDPC codes can be selected as an excellent coding scheme to achieve the highest reliability transmission. The objective in this paper is to develop a model for LDPC codes using simulation in Matrix laboratory language (MATLAB) on which BER calculations is carried out and to evaluate the performance, for different  $E_b/N_0$  levels, by varying the code-length, column weight, number of iterations, code-rate of the LDPC code.

**Keywords:** BER v/s  $E_b/N_0$ , LDPC, Mackay matrix.

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## I. INTRODUCTION

In recent years, there has been an increasing demand for efficient and reliable digital data transmission and storage systems. This demand has been accelerated by emergence of large-scale, high-speed data networks for the exchange, processing and storage of digital information in the commercial, governmental and military spheres. A merging of communications and computer technology is required in the design of these systems. A major concern of the system designer is the control of errors so that the data can be reliably reproduced.

To achieve reliable and high data transmission in modern communication systems, error correction coding (ECC) techniques are used usually combined with bandwidth-efficient modulation schemes. Especially, with effective iterative decoding algorithms, turbo codes and low-density parity-check (LDPC) codes are two powerful coding techniques [1].

### LDPC CODES

Low-density parity-check (LDPC) codes, originally invented by Gallager in 1962 is a linear block code whose parity check matrix is composed of '0' elements dominantly. But, since realization was regarded to be impossible in those days, it had been forgotten for a long time until Mackay rediscovered it in 1996. LDPC code shows good error correcting capability with iterative decoding by the sum-product algorithm. One of the recent topics in coding theory today are LDPC codes as an ideal candidate for next generation communication systems like wireless, satellite, magnetic recording channels and fiber optical applications.

In information theory, a low-density parity-check (LDPC) code is a linear error correcting code, a method of transmitting a message over a noisy transmission channel, and is constructed using a sparse bipartite graph. LDPC codes are capacity-approaching codes, which means that practical constructions exist that allow the noise threshold to be set very close (or even arbitrarily close on the BEC) to the theoretical maximum (the Shannon limit) for a symmetric memory-less channel. The noise threshold defines an upper bound for the channel noise, up to which the probability of lost information can be made as small as desired. Using iterative belief propagation techniques, LDPC codes can be decoded in time linear to their block length [2].

LDPC codes are also known as Gallager codes, in honor of Robert G. Gallager, who developed the LDPC concept in his doctoral dissertation at the Massachusetts Institute of Technology in 1960 [3].

LDPC belongs to block codes and are represented by a parity-check matrix H, where H is a binary matrix that, must satisfy

$$cH^T = 0$$

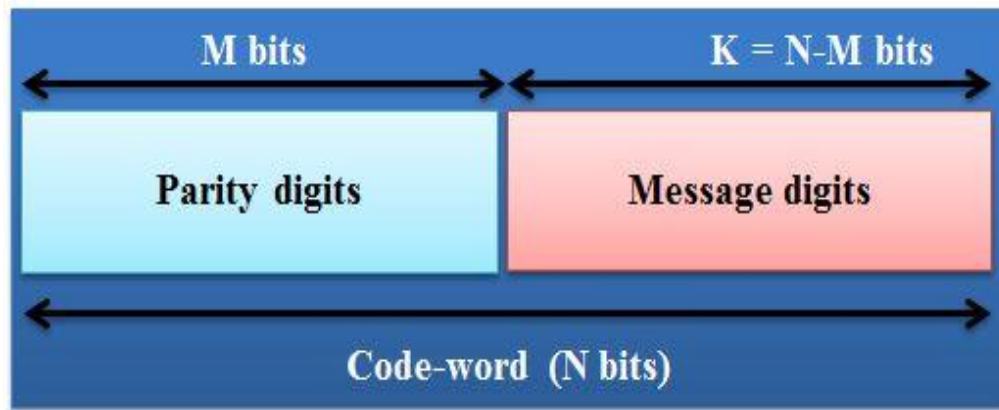


Fig. 1: General representation of LDPC (Block codes)

The parity check matrix H is sparse, i.e., for LDPC, the number of non-zero element is rather low compared with zero elements, which presents a character of low density.

In check matrix H, each column represents a coded bit while each row corresponds to a check sum. The number of non-zero elements for each column is defined as column weight ( $w_c$ ) and similarly row weight refers to the number of non-zero elements for each row ( $w_r$ ) [4].

Here is an example of check matrix H in eq. (1)

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ \dots & & & & & & & & & & & 1 \end{bmatrix}$$

This is a check matrix whose column weight is three and row weight is six. For a matrix H of  $M \times N$ , LDPC codes can be marked as  $(N, w_c, w_r)$  with N standing for the block size, and  $w_c$  and  $w_r$  standing for column weight and row weight respectively. Generally  $w_c \geq 2$  and  $w_r > w_c$  has to be satisfied [4].

### MacKay Codes

MacKay rediscovered Gallager's work in [6] and showed that these codes could also achieve near capacity performance like the well-known Turbo codes [7].

MacKay suggests that encoding be performed by using the generator matrix G obtained through Gaussian elimination from H. This method is not efficient because even though the parity-check matrix is sparse the generator matrix is generally not. Therefore, the encoding complexity of long block length codes generated in this manner would be high [5].

In this method, columns in H are generated from left to right until the whole check matrix produced. Column weight can be ensured to satisfy the demand as premise and the position of non-zero elements is randomly chosen between rows as long as the maximum assigned row weight does not exceed. Reset of H or cancelation and reset of some rows from right to left in the matrix occurs when the row weight cannot meet requirements when setting the last column [4].

In the following the procedure for generating H in this way is illustrated. Assume that H is 9\*12,  $w_r = 4$  and  $w_c = 3$ , given by eq. (2)

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \dots (2)$$

When setting the 11th column, it is found out that there are five rows (2, 4, 5, 6 and 9) not satisfying demands, with row weight less than 4. Hence, 1s should be placed in some of these rows for the certain column and in this case 2nd, 4th and 6th rows are selected.

## II. SYSTEM MODEL

The LDPC system model consists of transmitter, AWGN channel and a receiver.



Fig. 2: LDPC system model

### Transmitter of LDPC system

The transmitter of the LDPC system consists of creating LDPC matrix, eliminating length-4 cycle, generating parity check matrix using LU factorization on LDPC matrix. The generated data is then encoded using the parity check matrix. BPSK technique is used for modulation.



Fig. 3: Transmitter of LDPC system

### AWGN Channel

High data rate communication over additive white Gaussian noise channel (AWGN) is limited by noise. The received signal in the interval  $0 \leq t \leq T$  may be expressed as

$$r(t) = s_m(t) + n(t)$$

Where  $n(t)$  denotes the sample function of additive white Gaussian noise (AWGN) process with power-spectral density.

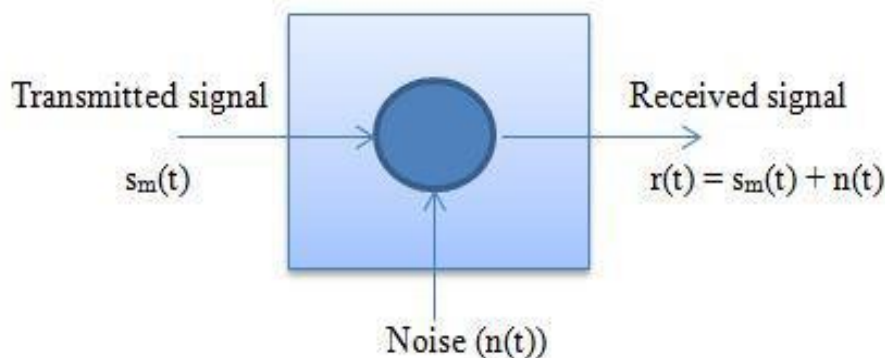


Fig 4: Model for received signal passed through AWGN channel

### III. SIMULATION

The simulation results are presented to analyze BER for various code-lengths for LDPC system using MATLAB. Plots between BER and  $E_b/N_0$  for digital modulation technique BPSK are plotted and its comparisons are carried out.

The design parameters are derived according to the system requirements. The design parameters for an LDPC system are as follows:

- **Code-length:** The LDPC code with large code-length gives the better BER results. But, at the same time, this also increases the encoding complexity.
- **Code-rate:** Higher code-rates works better in good channel conditions. However, for worst channel conditions lower code-rates give the better results.
- **Column Weight:** Lower the column weight, more the sparse-ness of the parity-check matrix; which provides the better results.
- **Iteration:** Large number of iterations gives improved results.
- **Encoding:** MacKay encoding is used using random construction of LDPC matrix. The parity-check matrix is created using LU factorization.

Table 1: General Parameters consider in simulation

Parameters	Values
Code-Length	5290
Code-Rate	0.8733
Iterations	5,10, 20, 25,50
Frames	10
Column Weight	2, 3, 4
Row Weight	4
Modulation	BPSK
Coding	LDPC

**A. Bit error rate curve for un-coded BPSK vs LDPC coded BPSK**

N = 5290, M = 670, Rate = 0.8733, Column-weight = 2, Iteration = 20

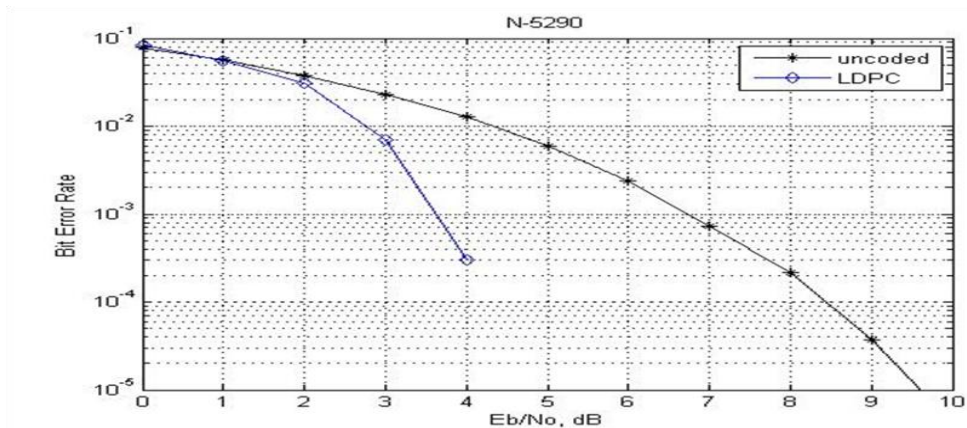


Fig. 5: Demonstrates plot of BER against Eb /No for BPSK

**B. Bit error rate curve for LDPC codes for different Column weights**

N = 5290, M = 670, Rate = 0.8733, Column-weight = 3, 4, Iteration = 20, BPSK

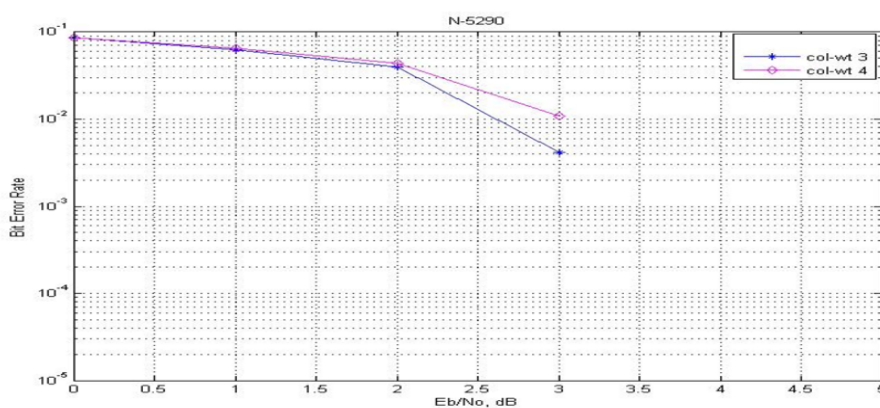


Fig. 6: Demonstrates plot of BER against Eb /No for LDPC with different column weights

**C. Bit error rate curve for LDPC codes for different number of iterations**

N = 5290, M = 670, Rate = 0.8733, Column-weight = 2, Iteration = 5, 25, Mod- BPSK

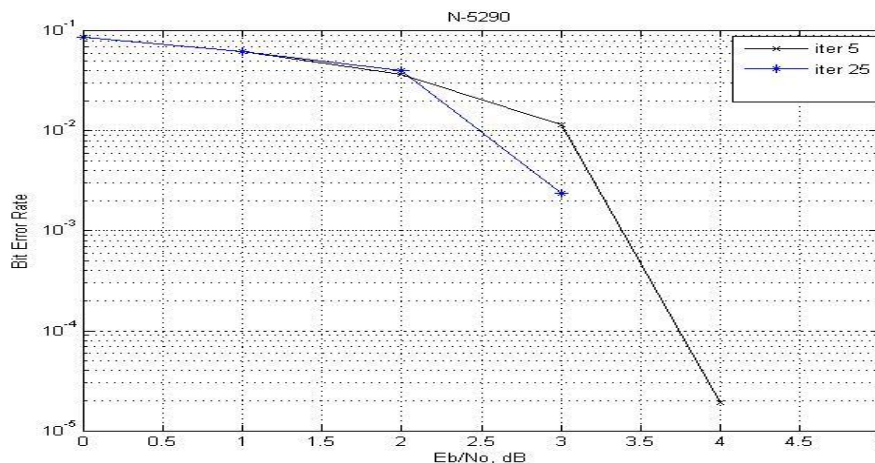
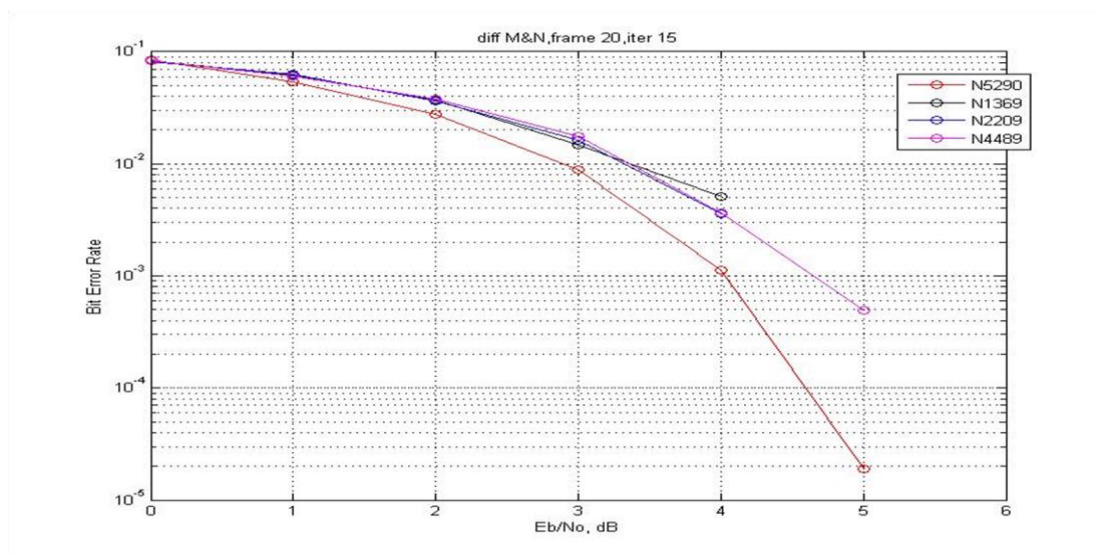


Fig. 7: Demonstrates plot of BER against Eb /No for LDPC with different number of iterations



**D. Bit error rate curve for LDPC codes for different code-lengths**

- $N = 5290, M = 670, \text{Rate} = 0.8733,$
- $N = 1369, M = 109, \text{Rate} = 0.9204$
- $N = 2209, M = 185, \text{Rate} = 0.9163$
- $N = 4489, M = 331, \text{Rate} = 0.9263$
- Column-weight = 3, Iteration = 15, Frame = 20, Modulation = BPSK



**Fig. 8: Demonstrates plot of BER against  $E_b/N_0$  for LDPC with different code-lengths**

#### IV. CONCLUSION

In this work, LDPC codes are constructed using MacKay encoder. The modulation technique used is BPSK and the modulated LDPC signal was transmitted over the AWGN channel. The code designed by this structure shows better performance up to  $10^{-4}$  to  $10^{-5}$  of BER in our simulation results. The work shows a property such that there is less degradation in randomly generated parity-check matrix when the codeword length becomes longer. Better performances are achieved for lower column weight in parity-check matrix for the LDPC code. For more number of iterations for message passing decoding algorithm, LDPC code gives better results. Moreover, the LDPC codes show better performance for lower code-rates for a particular code-length.

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